$$u=\int_{x_2}^{x}\int_{x_0}^{x}W\left(\frac{x}{y}\right)dx\,dx+cxy+w_2(y),$$

where W and w_2 are arbitrary functions in x/y and y, respectively, and c is an arbitrary constant.

II. **Solution by GEORGE W. HARTWELL, Columbia University, New York City.**

Let $\frac{\partial u}{\partial x}=p$. Then the equation will become

$$x\frac{\partial p}{\partial x}+y\frac{\partial p}{\partial y}=p.$$

Solving this by the method of Lagrange we have

$$p=y\,\phi\left(\frac{x}{y}\right)=\frac{y^2}{y}\,\phi\left(\frac{x}{y}\right).$$

Let $\phi\left(\frac{x}{y}\right)$ be the derivative of $\psi\left(\frac{x}{y}\right)$ with respect to $\frac{x}{y}$. Then

$$\frac{1}{y}\,\phi\left(\frac{x}{y}\right)=\frac{\partial}{\partial x}\,\psi\left(\frac{x}{y}\right).\quad \therefore\frac{\partial u}{\partial x}=y^2\frac{\partial}{\partial x}\,\psi\left(\frac{x}{y}\right).$$

Integrating, $u=y^2\,\psi\left(\frac{x}{y}\right)+\chi(y)$.

III. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Multiply the equation by x and let $x=e^v$, $y=e^w$.

Then $\frac{d^2u}{dv^2}+\frac{d^2u}{dvdw}=2\frac{du}{dw}$. Let $\frac{d}{dv}=D$, $\frac{d}{dw}=D'$. Then $D(D+D'-2)u=0$.

Let $u=e^{av+bw}$. $\therefore\frac{du}{dv}=au$, $\frac{du}{dw}=bu$.

$\therefore a(a+b-2)=0$, therefore, $a=0$, and $a=2-b$.

$\therefore u=\Sigma Ae^{bw}+e^{2v}\Sigma Be^{b(w-v)}=F(w)+e^{2v}f(w-v)=F(y)+x^2f(y/x)$,

where F and f are arbitrary.

Also solved by the Proposer.

MECHANICS.

197. **Proposed by WALTER D. LAMBERT, 416 B Street N. E., Washington, D. C.**

Suppose that a primary planet and its satellite revolve with uniform angular velocity in circular orbits in the same plane. What relation must hold between the radii of their orbits and their angular velocities in order

that the curve traced by the satellite shall be everywhere concave to the sun? Apply to the earth-moon system to prove that the moon's path is always concave to the sun.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let m=sun's mass, m_1=primary's mass, R=distance of primary from sun, r=distance of satellite from primary, v=velocity of primary around the sun at distance R, v_1=velocity of satellite around the sun at distance R, v_2=velocity of satellite around the primary at distance r.

Then $v^2/R : v_1^2/r = m : m_1$; $v : v_2 = \sqrt{m} : \sqrt{m_1}$; $v_2 : v_1 = \sqrt{r} : \sqrt{R}$.

$\therefore v : v_1 = \sqrt{(mr)} : \sqrt{(m_1 R)}$, or $v/R : v_1/r = \sqrt{(mr^3)} : \sqrt{(m_1 R^3)}$, the ratio of the angular velocities of primary and satellite in their respective orbits.

Hence, the path of the satellite will be looped, cusped, or direct throughout if

$$\sqrt{(m_1R^3/mr^3)} >=< R/r;\text{ or } m_1R >=< mr,\text{ or } m_1/m >=< r/R.$$

From these, we learn that the path of the satellite will be partly convex, just fail of being convex at perihelion, or be concave, if

$$m_1R^3/mr^3 >=< R/r;\text{ or } m_2R^2 >=< mr^2;\text{ or } m_1/m >=< r^2/R^2;$$
$$\text{or } \sqrt{(m_1/m)} >=< r/R.$$

For the earth and moon, $m/m_1 = 322,700 = (568)^2$, $R = 92,000,000$. Hence, if the moon were $92,000,000/322,700 = 285$ miles from the earth, it would travel in a cusped epicycle. If $92,000,000/568 = 162,000$ miles from the earth, the epicycle would be convex. As the actual distance is 238,828 miles, it is always concave.

Also solved by A. H. Holmes.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

141. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Given that the highest factor of a prime p contained in $m!$ is p^{m-s}; find general expressions involving p and m and s, from which, when a solution is possible, m can be determined when s is a given integer and p is a given prime. Is it then possible in any case to have more solutions than one?

No solution has been received.

142. Proposed by DR. L. E. DICKSON, The University of Chicago.

Let n be an integer >1 and set $p=n(n-1)+1$. Required n integers whose $n(n-1)$ differences are congruent (modulo p) to the numbers $1, 2, \ldots, p-1$. Exhibit at least for $n=3, 4, 5$, all inequivalent sets of solutions where a set $a_1, a_2, \ldots, a_n$ is called equivalent to the set $m(a_1-d), m(a_2-d), \ldots, m(a_n-d)$, for any integers m and d (m not divisible by p).